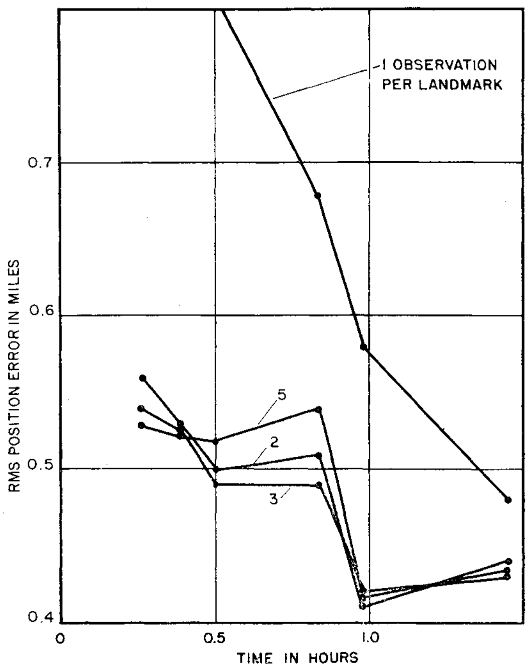

Fig. 3 Landmark uncertainty = 0.5 mile.

As a nominal trajectory, a 100-mile circular orbit around the earth from Cape Kennedy was selected. It was assumed that one landmark could be observed from one to five times at each of the following six areas of the earth: 1) West Coast of Africa, 2) Central Africa, 3) East Coast of Africa, 4) West Coast of Australia, 5) East Coast of Australia, and 6) Mexico.

Statistical data that were held constant are as follows:

1) The initial correlation matrix

$$E_0 = E_0{}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{pmatrix}$$

This matrix corresponds to position and velocity injection errors of 1 mile and 10 mph in any three mutually orthogonal directions.

2) The mean squared instrument error

$$\langle \alpha^2 \rangle = 10^{-6} \text{ rad}^2$$

Figure 1 shows the results obtained with perfect landmark knowledge for various values of the number of observations per landmark. Shown are the rms position errors occurring at the conclusion of all observations of each landmark. The errors are substantially larger at intermediate times, but are omitted for convenience. Similar results are illustrated in Figs. 2 and 3 for landmark position uncertainties of 0.25 and 0.5 miles, respectively.

It is seen from these results that, the larger the landmark uncertainty, the less worthwhile it is to make additional observations of the landmark. In fact, it appears that a landmark should be observed exactly twice unless its position is known quite accurately.

The results of the preceding study indicate the effectiveness of this method of orbital navigation. An accurate estimate of the trajectory of the spacecraft can be obtained by merely a few observations of landmarks on the surface of the planet.

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Chemical and Nozzle Flow Losses in Hypersonic Ramjets

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Introduction

Because of the small differences that exist between the large values of inlet and exhaust momentum, the performance of hypersonic ramjets is extremely sensitive not only to recombination losses, but also to the exhaust nozzle divergence and frictional losses. For example, a very long nozzle will insure equilibrium flow, but large friction losses will result; on the other hand, rapidly divergent nozzles may tend to freeze the flow and minimize the friction losses, but the divergence loss will be increased. The problem reduces to finding the optimum tradeoff among all the losses. The objective of this study was to investigate the tradeoff among these losses for a supersonic combustion ramjet flying at a Mach number of 15 and an altitude of 150,000 ft.

Analysis

For the supersonic combustion ramjet, the nozzle entrance conditions are dependent upon the inlet performance and mode of combustion. The following assumptions were made concerning the processes: 1) the value of inlet kinetic energy efficiency η_k (referred to stagnation conditions) is constant at 0.990; 2) the inlet diffuser velocity ratio V_2/V_1 is a variable parameter; 3) hydrogen and air at stoichiometric proportions are burned in a constant area duct with no loss in combustion efficiency; 4) the effective fuel velocity for performance calculations is 6400 fps including losses due to friction and blockage drag; 5) the inlet capture area A_1 is 100 ft^2 and the nozzle exit area A_4 is 150 ft^2; 6) the flow entering the nozzle is in equilibrium and one-dimensional; 7) the flow through the nozzle is one-dimensional; and 8) axisymmetric conical nozzles are used for calculating all losses.

Analytical procedures for calculating the extent of recombination of complex chemical reactions in adiabatic one-dimensional flow systems have been described in published form elsewhere.[1-4] The equations and computational procedure,

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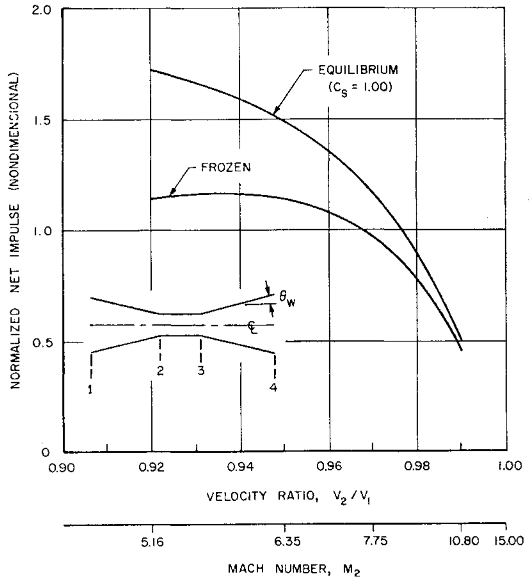

Fig. 1 Effect of combustion chamber entrance velocity on ramjet performance for flight Mach number $M_1 = 15$ at 150,000 ft.-alt. The inlet area to exhaust ratio $A_4/A_1 = 1.5$ and the kinetic diffuser efficiency $\eta_k = 0.990$

along with the reaction mechanism for H_2 air utilized in this study, have also been published.[1]

For the conical nozzles used in this study, a divergence loss of $(1 + \cos\theta_w)/2$ was applied to the exit vacuum thrust in the absence of friction (θ_w is expansion half-angle). Previous studies[5] indicate that the vacuum thrust loss does not differ significantly from the source flow assumption. A method for estimating the skin friction proposed by Reshotko and Tucker[6] was used which agreed well with flat-plate, adiabatic wall data.[7] With a reference flat-plate length of 50 ft, along with a constant wall temperature of 2000°R, used to calculate the Reynolds number at the entrance to the nozzle, an almost constant level of friction coefficient along the nozzle wall was obtained. A number of calculations of this type for various values of V_2/V_1 indicated that a constant value of skin-friction coefficient appeared justified for any given velocity ratio (V_2/V_1). The friction coefficients C_f used for this study varied from 0.0023 to 0.0033 when the corresponding velocity ratio V_2/V_1 varied from 0.92 to 0.99.

The techniques utilized in the parametric study of nozzle performance proceeded as follows. Several discrete values of the velocity ratio (V_2/V_1) from 0.99 to 0.92 were assumed. At each velocity ratio the nozzle wall angle was varied, and the losses due to friction and divergence were estimated as previously described; recombination losses due to a nonequilibrium expansion were estimated assuming a one-dimensional expansion.

Results and Discussion

The two limiting cases, equilibrium and frozen flow, may be used to indicate in a general manner the effect of the velocity at the entrance to the combustor on ramjet performance as shown in Fig. 1. The performance for equilibrium nozzle flow assumed no losses in expanding to the exit area indicated previously. Since the value of inlet kinetic energy efficiency was assumed constant for this study, the ramjet net impulse is always improved by lowering the combustor velocity if equilibrium nozzle flow is assumed. Although a greater percentage of dissociated species occurs at the nozzle entrance with lower values of V_2, there is no net loss if the dissociation energy is isentropically reconverted to kinetic energy in the nozzle, whereas, the combustor total pressure losses are decreased by combustion at a lower Mach number.

When the nozzle flow is frozen at the nozzle entrance (station 3) there is an optimum amount of diffusion as shown in Fig. 1. In this case, the benefits of burning at a lower Mach number are opposed by the losses incurred by not recovering the increased amount of dissociation energy.

A typical result, which shows the variation of all of the previously mentioned losses as a function of the nozzle wall expansion angle, is shown in Fig. 2. Similar trends were obtained for all of the cases studied. It can be seen from Fig. 2 that the optimum value of nozzle thrust efficiency C_s obtained by minimizing the friction and divergence losses occurs within a nozzle wall angle between 10° and 15°. If the nonequilibrium losses had a large effect on the optimum wall angle, the peak value of C_s would be displaced toward much lower wall angles. Because no significant displacement is noted in Fig. 2, it appears for the flight conditions assumed that the friction effects far outweighed the importance of minimizing the nonequilibrium losses. In other words, the low wall angles necessary to insure equilibrium flow produced excessive friction losses. In all cases studied, optimum performance was obtained by minimizing the combination of

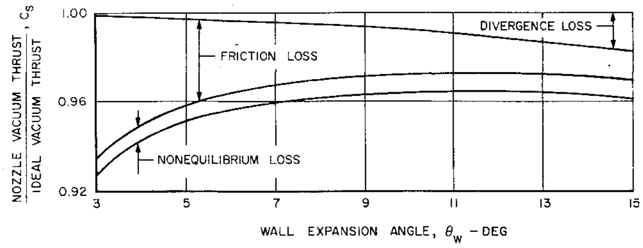

Fig. 2 Effect of nozzle wall expansion angle on losses at a diffuser velocity ratio V_2/V_1 equal to 0.970 for the ramjet configuration and for conditions of Fig. 1.

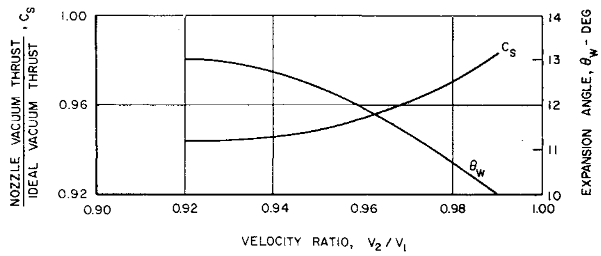

Fig. 3 Effect of diffuser velocity ratio V_2/V_1 on optimized values of nozzle thrust efficiencies C_s and wall expansion angles θ_w for conditions of Fig. 1.

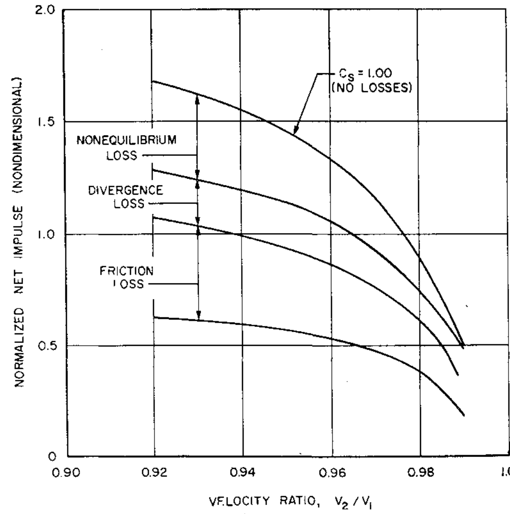

Fig. 4 Comparison of impulse for nonequilibrium, divergence and friction losses with varying velocity ratio V_2/V_1 and for the expansion wall angle $\theta_w = 12°$ and flight conditions of Fig. 1.

friction and divergence losses and accepting a frozen (or nearly frozen) flow. The optimum values of C_s obtained are shown in Fig. 3 along with the optimum value of nozzle wall angle as a function of velocity ratio. These values of C_s were obtained from studies similar to those that yielded the results shown in Fig. 2.

The optimum values of C_s shown in Fig. 3 were converted to a normalized net impulse and plotted as shown in Fig. 4 for a wall angle of 12. The lowest line shown represents the maximum computed impulse. Also a breakdown of the losses that produced the maximum impulse for each velocity ratio is indicated. The high level of these losses is apparent.

In summary, these studies suggest that, for conical nozzles, nonequilibrium losses are less important than wall friction for the flight conditions assumed. To investigate the effects of variations along a flight trajectory, further nozzle optimization studies of this type should be conducted for conical nozzles as well as other shapes. Because the results obtained depend on the skin friction utilized, experimental verification of nozzle skin-friction coefficients and means to reduce those coefficients should also be investigated.

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Implementation of an Optimal Adaptive Guidance Mode

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Introduction

THE guidance technique employed by multistaged and clustered space vehicles generally must accommodate a wide range of disturbances in the state vector of the vehicle during the accomplishment of its mission. Several of these guidance techniques depend upon the generation of trajectories optimized with respect to some specified criterion. Many methods exist for the generation of such trajectories in such a way that the possible disturbances are included. This leads to a tabulation of values of the optimal control laws for a specific mission. The tabulation of optimal control laws and their approximation by easily generated functional forms for use in actual control systems is referred to as a "flooding technique" by Kipniak,[2] and, in a more specialized form, as approximating a "statistical model" by Schmieder and Braud.[4] The digital computers with limited memories now being used for the guidance of space craft allow immediate realization of this form of optimal adaptive guidance (O.A.G.).

Method of Implementation

N sets of initial conditions are selected so as to encompass possible disturbances in the state of the vehicle during its mission and yet allow the successful completion of that mission in an optimal fashion. The optimized control laws are derived from the numerical solution of the N two-point boundary-value problems. The data for any one of the N trajectories are given in the form

$$u_i = [c_1(\mathbf{X}_i), c_2(\mathbf{X}_i), c_3(\mathbf{X}_i), \mathbf{X}_i]$$

where

$$\mathbf{X} = (X_1, X_2, \ldots, X_m)$$

$$\mathbf{X}_i = (X_{i1}, X_{i2}, \ldots, X_{im})$$

The subscript i refers to the ith time point on the trajectory and $c_K(\mathbf{X}_i)$ is the value of the Kth control function (K = 1, 2, 3) at the point $\mathbf{X}_i$. The X_v's ($v = 1, \ldots, m$) are the measurable state variables of the vehicle; e.g., position and velocity coordinates, time, and perhaps thrust divided by mass flow.

The present note reduces the problem of implementing O.A.G. to one of obtaining approximations of the c_K with linear or rational combinations of certain M basis functions, b_j, of the state variables

$$c_K \doteq \sum_{j=1}^{M} \alpha_{Kj} b_j \tag{1}$$

or

$$c_K \doteq \Big(\sum_{j=1} \alpha_{Kj} b_j\Big)\Big(\sum_{j} \beta_{Kj} b_j\Big)^{-1} \qquad K = 1,2,3 \tag{2}$$

The values of the basis functions b_j must be easily generated from values of the state variables $\mathbf{X}$. The coefficients α_{Kj} and β_{Kj} are fixed for each particular mission and stored in the memory of the onboard digital computer. Values of the control functions are calculated during the flight of the vehicle from the approximations (1) or (2). The interval between successive evaluations of the control functions depends on the mission and is limited by the response rate of the control hardware. Usually, for lunar and orbital missions, this interval will be well under 10 sec.

The special case where the basis functions b_j are monomials in the state variables has been considered in a number of reports.[4, 5, 7] This resulted in sufficiently accurate approximations to the control functions for practical use. A detailed error analysis of a particular mission using polynomial O.A.G. was given by Morgan.[3]

The effectiveness of this type of O.A.G. is directly related to the errors in the approximation of the c_K's. To determine the coefficients α_{Kj} and β_{Kj} of (1) and (2), a set of points u_i is selected for use in some numerical method of approximation. The magnitude and place of occurrence of the errors e_i are related to the u_i selected and to the form of the approximant (1) or (2). A third source of errors arises in the numerical method used to determine the unknown coefficients α_{Kj} and β_{Kj}.

When the forms of the approximants of the c_k's were polynomials, as in (1), a straightforward least-squares numerical method regularly failed to give accurate values of the α_{Kj} because of the associated system of ill-conditioned normal equations. Moreover, elimination of those basis functions b_j (monomials) that did not reduce the magnitude of the errors was difficult. These difficulties were resolved by the

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